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Full Length Research Paper

Modeling of periodic and stochastic component of monthly rainfall in Senegal

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Abstract:
Analysis of hydroclimatic variables is crucial for the elaboration of local sustainable development policies, especially in countries with a rain fed agriculture. Hence, modeling the rainfall behavior might be very helpful, particularly in designing strategies to struggle against the climate change effects. This study aims at seeking representative models that fit the monthly rainfall in Senegal. Thus, the simple Mann-Kendal trend test is applied to the raw available records in order to assess the trend significance. Autocorrelogram curves are analyzed to check periodic behavior of the series. Harmonic analysis is performed for the periodic component designing. The periodogram curve is involved to determine the number of significant harmonics in the fundamental period. The stochastic components of the monthly rainfall are modeled using appropriate autoregressive process, performed upon the remaining series after extracting deterministic components. The raingauges are chosen according to the three climate bands covering the territory of Senegal and all the time series ranges from 1970 to 2010. The Mann-Kendal test show that trends of the monthly rainfall series can be neglected in the modeling scheme. The autocorelogram analysis shows a periodic behavior of the whole series. The retained significant harmonics according to the periodogram analysis are of four at Saint Louis, Dakar, Kaolack and of three at Ziguinchor and Tambacounda. A first order autoregressive process has been identified to model the stochastic components. The exploratory analysis of the evolution in time of simulated and observed time series shows good correlation. Further, the comparison of statistic parameters of the series such the mean, the standard deviation, the coefficient of variation and of determination confirms the good skill of the proposed models. Therefore, models can be retained for any monthly rainfall data requirement in the corresponding zones.

Mots clés/Keyword: Senegal; Time series; Simulation; Harmonic analysis; Stochastic process

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1. Introduction

In West Africa, climate variability and change is seriously affecting agricultural activities. About Senegal, rain fed agriculture remains the most important survival activity and is mainly oriented towards cereal production such the rice and the millet that constitute the main food crop. However, in agricultural domains, rainfall time series are needed for sustainable development planning and decision making support (Paredes-Trejo at al. 2021). Then, studying hydroclimatic variables such as rainfall, evaporation, ground water level, river flow and wind speed is very important either for their deterministic factors understanding or for some hydrological studies requirements (Ndione et al. 2018; Diatta et al. 2020). In practice, the water specialists often face to data shortcomings and in some circumstances. They therefore have recourse to time series modeling for huge hydraulic and hydrological applications: reproducing longer hydrological variable according to applications requirements, forecasting hydrological events or restoring missing data (Bhakar, 2006;

Foufala-Georgiou et al., 1995, Zarei et al. 2020.a, Moghimi et al. 2020). However, the random behavior of the hydroclimatic variables makes the modeling of the time series through the classical fluids mechanics lows very difficult (Kottegoda, 1980; Fontin, 1987). Therefore mathematical schemes are widely used for the modeling of such variables (Moghimi et al. 2020). In the renowned mathematical modeling schemes, hydrological variables are assumed to depend on the whether parameters inducing deterministic components (periodic and trend) and a stochastic one (Kottegoda, 1980; Zakaria, 2011, 2014; Pandey et al., 2009; Mbaye et al. 2019, Zarei et al. 2021.b, Paredes-Trejo at al. 2021). Thus, hydrological time series is often divided into a sum of trend, periodic, stochastic components. The trend corresponds to the long-lasting change in time series relatively to the mean. The periodic component that can be seasonal in nature is induced by cyclical behaviors in the fundamental period (p). It is defined by similarity between ith and $(i \mp np)$th value of the time series. In the literature, mathematical time series modeling have been used in a large hydrological studies. Ndione et al. (2020) set up a stochastic scheme for daily rainfall ensemble forecasting used as HBV-Light input. Good performances have been obtained witch corresponding hydrological ensemble forecasting. Stojkovic et al. (2015) established a stochastic model to better estimate the annual mean flow series in the purpose of implementing a stochastic ensemble forecasting system for water resources planning and management. Valent et al. (2011) used a nonlinear model to produce a synthetic time series in order to describe and predict the nitrate concentrations of two rivers located in Eastern England. Dabral et al. (2014) used a steady time series modeling method for simulating pan evaporation with good performances according to some classical verifications methods. In the same vein, Pandey et al. (2009) modeled the black gram evapotranspiration for a forecasting purpose. In Ahaneku et al. (2014), synthetic monthly rainfall sequences have been produced on the basis of the moment analysis considering the dependency of the involved structures. Saada (2014) shows that the PARMA model is more appropriate for simulating monthly rainfall in arid zones such as the Saudi Arabia one. Zakaria et al. (2011) simulated cumulative rainfall over 25 years at Purajayain using the spectral analysis, the Fourier decomposition and a third-order autoregressive model. Bhakar et al. (2006) used harmonic analysis and a one order autoregressive process to simulate monthly rainfall at the Kota region. In this paper, representative statistic models for monthly rainfall in Senegal are set up which parameters are estimated using recorded monthly rainfall from the National Civil Aviation and Meteorological Agency of Senegal (ANACIM). The statistic models from this study are intended to help development agencies and the government in planning and managing the vital rainfall related activities, particularly in water resources management and agriculture. Six raingauges have been selected according to the three climate bands covering the Senegal: one in the warm desert band facing the sea (Saint-Louis), one other in the tropical savanna also close to the sea (Ziguinchor) and four in the large warm and semi-arid band of the Senegal area (Dakar, Kaolack, Bakel and Tambacouda). While, in the warm and semi-arid band, two of the raingauges are towards the sea (Dakar (DK) and Kaolack (KL)) and the two others are located inside the country far to the sea (Bakel (BK) and Tambacounda (TC)). Trend in time series is assessed by applying the Mann-Kendal (MK) test. Autocorrelogram plot is used to exhibit periodic features in the monthly rainfall. Exploratory analysis of the time series periodogram is carried out for estimating the significant harmonics to be taken into account in the periodic component modeling. An appropriate autoregressive processes are carried out to model the stochastic component. The reliability of models is verify using comparison of the evolution in time of series composed of observations against simulations. In addition, statistic characteristics of the data such as the mean, the standard deviation (Sd), the coefficient of determination (R2), the coefficient of variation (Cv) are analyzed.

2. Material and methods

2.1. Area of study

Senegal is a country located in the extreme part of western Africa, between the latitudes 12 ° 8 - 16 ° 41 N and longitudes 11 ° 21 - 17 ° 32 ° W. Its area is of about 196712km2. The inside climate is characterized by two seasons: a rainy season from June to October and a dry one ranging from November to May. The yearly rainy season rarely goes beyond four months (Ndione et al. 2017). The data used in this study are from the database of the National Civil Aviation and Meteorological Agency of Senegal (ANACIM) and are composed of monthly rainfall depth selected in the three climate zones of the World Coppen classification: the warm and desert climate band covering the Northern Senegal (Saint-Louis), the warm and semi-arid climate band covering central regions of Senegal (Dakar, Kaolack, Bakel and Tambacounda) and the tropical savanna climate in Southern Senegal (Ziguinchor). Time series recording period ranges from 1960 to 2010. Positions of the selected raingauges are shown in Fig. 1. The table 1 gives raingauges code, geographical position, data scale, time series span and the recording period. In additional to the map (Fig. 1), some hydroclimatic characteristics of the study area are shown in Table 2 for more information. The concerning additional informations are composed of the Mean of the Maximum Temperature of the Dry Season (MmaxTDS) and the one of the Rainy Season (MmaxTRS); Mean of the Minimum Temperature of the Dry Season (Mmin TDS) and the one of the Rainy Reason (Mmin TRS); Mean of the Maximum of Air Moisture of the dry season (Max. AMDS) and the one of the Rainy Season (Max. AMRS); Mean of the Minimum of Air Moisture of the Dry Season (Min. AMDS) and the one of the Rainy Season (Min. AMRS) and finally, the Mean Monthly Evaporation of the Rainy Season (MMERS), and the one of the Dry Season (MMEDS)

Tale 1. – Raingauges and informations regarding the data

Stations	Code of stations	Location		Data	Period
		Longitudes	Latitudes		
St-Louis	38004500	-16.05°	16.05°	Monthly rainfall	1970-2010
Bakel	38007200	-12.45°	14.90°	Monthly rainfall	1970-2010
Dakar	38008100	-17.5°	14.73°	Monthly rainfall	1970-2010
Kaolack	38009700	-16.07°	14.13°	Monthly rainfall	1970-2010
Ziguinchor	38013700	-16.27°	12.55°	Monthly rainfall	1970-2010
Tambacounda	38011300	-13.68°	13.77°	Monthly rainfall	1970-2010

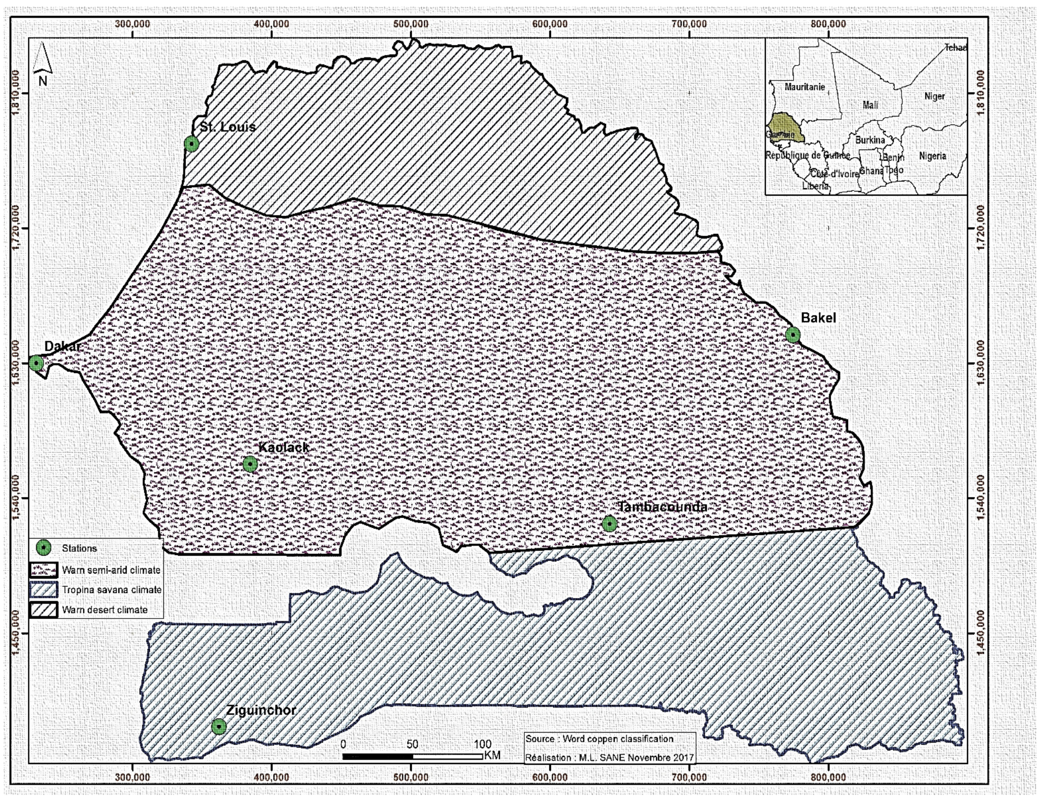


Figure 1 : **Position of the raingauges through the study area**

Tale 2. – Additional hydroclimatic characteristics of the study area

Stations	M_{max}.TDS (°C)	M_{max}.T RS (°C)	M_{min}.T DS (°C)	M_{min}.T RS (°C)	Max.AM DS (%)	Max.AM RS (%)	Min.AM DS (%)	Min.AM RS (%)	MMCE DS (mm)	MMCE RS (mm)
S-L	31.8	31.9	17.7	24.2	81.9	92.71	34.0	61.3	5.2	2.9
Bakel	38.2	36.6	21,9	24,5	45,3	85,6	18,6	46,4	11,2	11,2
Dakar	26.1	29.9	19.3	24.6	90.6	89.9	55.5	67.7	3.0	2.5
Kaolack	37.7	34.9	19.7	24.2	67.5	93.1	22.50	53.0	6.8	2.9
Ziguinchor	36.3	32.8	20.2	21.5	88.84	97.1	30.1	62.12	3.9	1.6

2.2. Time series modeling

Hydrometeorological variables are characterized by deterministic and stochastic features of which simultaneous estimating is unstraightforward (Mbaye et al. 2017). In practice, mathematical schemes are proposed for reproducing such variables up to the intrinsic behaviors of the real data. In the literature, scientists agree with a mathematical scheme in which time is assumed to be decomposable into deterministic and stochastic components (Kottegoda 1980; Zakaria 2011; Bhakar et al. 2006; Jhajharia et al. 2014). However, is important to keep in mind that the different components are treated separately in the modeling process. This paper aims at modeling the monthly rainfall in Senegal. Then, an observed time series referred to as X_t is divided into sum of three functions representing within retained time series components ((Dabral et al. 2014; Kuldeep et al. 2009). The additive form of the time series X_t is represented as follow:

$$X_t = \mathcal{T}_t + \mathcal{P}_t + \xi_t \quad (t=1,2,\ldots\ldots\ldots..N) \qquad \text{[Eq. 1]}$$

$\mathcal{T}_t$: trend component; $\mathcal{P}_t$: periodic component; ξ_t : stochastic component; N: number of observations.

2.2.1. Trend analysis

Trend in time series characterize the direction of the global evolution of the variable without considering the short-term fluctuations (Kottegoda 1987; Jhajharia 2014). However, an absence of trend in a time series doesn't mean that the data are stationary. Trend effect in hydroclimatic time series may be induced by deforestation effects, perturbation in hydrological systems or by the persistence of the climate change. In this study, trend is assessed applying the commonly used Mann-Kendal test. The null hypothesis H_0 of trend free is accepted when the p-value goes beyond 0.05. The test skills are detecting the significance of existing trend and giving the trend direction without linearity assessment (Khaliq et al. 2009; Ndione et al. 2018). The method is based on a statistic variable noticed by S with is defined according to the difference $x_i - x_j$ between all pairs of the time series $(x_{i,j})$. A negative value of the statistic indicates a decreasing trend, while a positive value indicates an increasing one (Önöz et al. 2002; Kottegoda 1980; Tesemma et al. 2010; Luo et al. 2008; Yue et al. 2004; Ndione et al. 2018).

$$S = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} sgn(xj \quad xi) \qquad \text{[Eq. 2]}$$

$$\text{with} \begin{cases} sgn(xj - xi) = 1 \text{ if } (xj - xi) > 0 \\ sgn(xj - xi) = 0 \text{ if } (xj - xi) = 0 \\ sgn(xj - xi) = -1 \text{ if } (xj - xi) < 0 \end{cases} \qquad \text{[Eq. 3]}$$

Then, the S is assumed to be independent and normally distributed with zero mean and variance given by Eq. (4)

$$Var(S) = \frac{N(N-1)(2N+5)}{18} \qquad \text{[Eq. 4]}$$

Hence, the standardized statistic variable Z of Mann-Kendall's S is defined as follow:

$$\begin{cases} Z = \frac{(S-1)}{\sqrt{var(S)}} \text{ if } S > 0 \\ Z = 0 \text{ if } S = 0 \\ Z = \frac{(S-1)}{\sqrt{var(S)}} \text{ if } S < 0 \end{cases} \qquad \text{[Eq. 5]}$$

2.2.2. Harmonic analysis

Periodic component such as seasonal signals in time series are deterministic in nature. In this paper, the periodic component is considered as the seasonal behavior of the studied monthly time series. In hydrological variables, seasonality is assumed being mainly induced by the rotation and revolution of the earth (Kottegoda 1980). This component is modeled using the harmonic analysis through the Fourier decomposition of the raw time series. The modeling begins on checking existence of periodic behavior in the raw time series by the analysis of the corresponding autocorrelogram which is reputed to be periodic feature keeper. The periodic component of the time series noticed by P_t is formulated as follow.

$$P_t - \mu + \sum_{i=1}^{L} A_i \sin\left[\frac{2\pi t}{p} i + \varphi_i\right] \qquad \text{[Eq. 6]}$$

μ: mean of the time series; L : the maximum harmonics in the fundamental period; $L = n/2$ if the n number of observations is even and L= (n − 1)/2 if it is odd; A_i : amplitude of the i^th harmonic; p : fundamental period; $\lambda_i = p/i$: wave length of harmonics; φ_i : harmonic phase (Kottegoda 1980). Considering monthly rainfall, the fundamental period is 12 months. The mean monthly rainfall of a set of τ months of the time series is given by the following equation.

$$m_\tau = \frac{n}{p} \sum_{i=1}^{n/p} x_{\tau+p(i-1)} \qquad \text{[Eq. 7]}$$

Where n/p is the number of years in the sequence of the time series. The mathematical expression of harmonics is obtained by transforming Eq.6 such that:

$$P_t = \mu + \sum_{i=1}^{p/2} a_i \sin\left[\frac{2\pi t}{p} i\right] + \sum_{i=1}^{p/2} b_i \cos\left[\frac{2\pi t}{p} i\right] \qquad \text{[Eq.8]}$$

The i^th harmonics h_i is given by the following equation.

$$h_i = \sum_{i=1}^{p/2} a_i \sin\left[\frac{2\pi t}{p} i\right] + \sum_{i=1}^{p/2} b_i \cos\left[\frac{2\pi t}{p} i\right] \qquad \text{[Eq. 9]}$$

The variance of a single harmonic h_i is given by the underlying equation.

$$\sigma^2(h_i) = \frac{\hat{a}_i^2 + \hat{b}_i^2}{2} \qquad \text{[Eq.10]}$$

Estimated parameters $\hat{\mu}$, $\hat{a}_i$ and $\hat{b}_i$ of the model scheme are obtained using the least square method. These estimates are given by Eq. (11), (12), (13) and (14) respectively.

$$\hat{\mu} = \frac{1}{p}\sum_{\tau=1}^{p} m_{\tau} \quad \text{[Eq.11]}$$

$$\hat{a}_k = \frac{2}{p}\sum_{\tau=1}^{p} m_{\tau}\sin\left(\frac{2\pi k\tau}{p}\right) \quad k = 1,\ldots,\frac{p}{2}-1 \quad \text{[Eq.12]}$$

$$\hat{b}_k = \frac{2}{p}\sum_{\tau=1}^{p} m_{\tau}\cos\left(\frac{2\pi k\tau}{p}\right) \quad k = 1,\ldots,\frac{p}{2}-1 \quad \text{[Eq.13]}$$

$$\begin{cases} \hat{a}_{p/2} = 0 \\ \hat{b}_{p/2} = \frac{1}{p}\sum_{\tau=1}^{p} m_{\tau}(-1)^{\tau} \end{cases} \quad \text{[Eq.14]}$$

2.2.3. Estimation of the significant harmonics

Significant harmonics to be considered in the modeling process is obtained through the periodogram analysis. The periodogram is constituted by the plot of the variances $\sigma^2(h_i)$ against their i corresponding ranks. The total harmonics to be considered in the model is reduced to the consecutives ones which contribution to the total variance is significant (Bhakar et al. 2006; 174 Fontin 1987; Kottegoda 1980; Jhajharia et al. 2014).

2.2.3. Modeling of the stochastic component

Stochastic component in time series is characterized by irregular and random fluctuations representing additional features upon deterministic components that can't be exhaustively handled by the known physical and deterministic approaches; thus, mathematic tools are involved (Kottegoda 1980; Fontin 1987; Jhajharia 2014). However, in time series modeling, assumed random component may include deterministic features (Bhakar et al. 2006). The stochastic component of the monthly rainfall series is referred to as ξ_t in Eq.1 and is designed using appropriate autoregressive process $AR(p)$. This mathematical process allows isolating the strictly random part η_t from the remaining time series obtained after extracting the deterministic components from the raw data. The strictly random part of the time series is called noise of the time series. The autoregressive process is defined as:

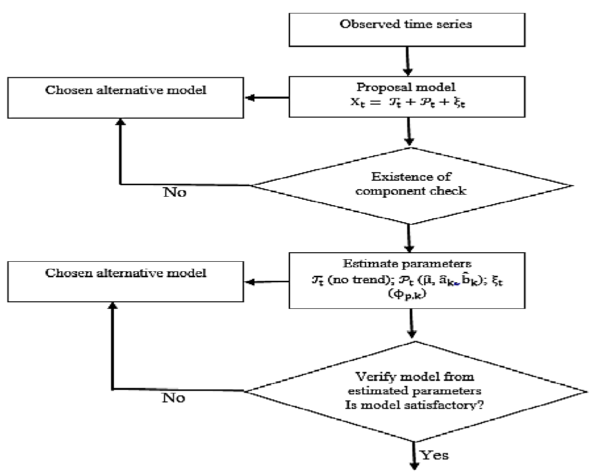
$$\xi_t = \sum_{k}^{p} \phi_{p,k}\xi_{t-k} + \eta_t \quad \text{[Eq. 15]}$$

$\phi_{p,k}$ (k = 1, 2,….,p): parameters of the autoregressive process and η_t the noise of the model The $\phi_{p,k}$ parameters are estimated using Eq.16 and 17 and are obtained by solving the Yule Walker equations defined as follow:

$$\phi_{p,p} = \left(\rho_p - \sum_{k=1}^{p-1}\phi_{p-1,k}\,\rho_{p-k}\right)/\left(1 - \sum_{k=1}^{p-1}\phi_{p-1,k}\,\rho_k\right) \quad \text{[Eq 16]}$$

$$\phi_{p,k} = \phi_{p-1,k} - \phi_{p,p}\phi_{p-1,p-k} \quad \text{with } k = 1,\ldots,p-1 \quad \text{[Eq 17]}$$

Further, a flowchart is proposed as summarize of the modeling process described in this section



3. Results and discussion

3.1. Statistical characteristics of the monthly rainfall

The mean, the standard deviation and coefficient of variation of all the monthly rainfall time series are represented in Table 3. At first, analysis of the means, the standard deviations and the coefficients shows that the rainfall decreases from South to North and from East to West. Accordingly, the standard deviations naturally increase in the same direction. The coefficient of variation generally decreases as the monthly rainfall increases. Consequently, the dispersion around the rainfall mean is more important in northern Senegal. The rainfall variability is the more visible in the warm desert and warm semi-arid bands. In addition, the mean monthly rainfall show that the rainfall in Senegal is constituted by two seasons: dry season from November to May and rainy one ranging from June to October. The above mean monthly rainfall show that the maximum occurred in August for all raingauges except Saint-Louis, where the maximum covering 37.18 % of the total rainfall is observed in September. Months with lower rainfall are also shown in Table 3.

Quota of rainfall in August are of 25.94 % at Bakel, 40.94 % at Dakar, 38.43 % at Kaolack, 32.57 % at Ziguinchor and 29.32 % at Tambacounda

Table 3. – Statistic characteristics (Mean, Standard deviation and coefficient of determination) of the observations

Stations \ Mois		Jan.	Feb.	Mar.	Apr	May	Jun	Jul	Aug	Sept.	Oct.	Nov.	Dec.	Mean
St-Louis	Mean	2.66	1.52	0.19	0.06	0.17	6.39	39.58	82.91	92.5	18.83	0.32	3.67	20.73
	Sd	10.79	3.67	0.53	0.27	0.57	9.50	44.09	42.67	68.76	30.33	1.06	16.3	
	Cv	4.05	2.41	2.78	4.5	3.35	1.4	1.11	0.51	0.74	1.61	3.31	4.44	
Bakel	Mean	0.34	1.06	0.54	0.24	6.2	46.7	131.0	177.8	136.1	22.8	1.37	0.46	43.72
	Sd	1.61	4.76	3.08	1.16	11.94	30.18	62.68	81.84	61.26	21.26	4.85	1.42	
	Cv	1.79	4.49	5.7	4.83	1.92	0.64	0.02	0.46	0.45	0.93	3.54	3.08	
Dakar	Mean	2.52	0.39	0.03	0.00	0.04	8.89	53.26	142.15	118.90	19.90	0.80	0.33	28.93
	Sd	9.68	0.95	0.10	0.00	0.14	18.01	46.38	75.75	61.74	25.86	3.33	0.37	
	Cv	3.84	2.43	3.33	0.00	3.50	2.02	0.87	0.53	0.51	1.29	4.16	1.12	
Kaolack	Mean	1.20	1.06	0.09	0.00	2.05	37.26	119.58	225.11	153.66	43.50	1.56	0.71	48.81
	Sd	5.01	3.88	0.31	0.00	7.32	35.49	62.39	83.12	65.89	41.55	6.77	2.57	
	Cv	4.17	3.66	3.44	0.00	3.57	0.95	0.52	0.36	0.42	0.95	4.33	3.61	
Ziguinchor	Mean	0.21	0.22	0.03	0.00	3.85	95.51	303.31	393.78	316.36	91.38	3.57	0.69	100.74
	Sd	1.12	0.81	0.17	0.00	6.88	65.46	99.72	136.16	99.78	52.03	10.40	2.55	
	Cv	5.33	3.68	5.6	0.00	1.73	0.68	0.32	0.34	0.31	0.56	2.91	3.69	
Tambacounda	Mean	0.04	0.31	0.29	0.50	17.74	87.93	178.86	210.14	161.00	56.75	2.80	0.45	59.73
	Sd	0.13	0.38	0.82	2.61	22.71	43.02	66.75	83.60	72.62	48.43	10.00	1.5	
	Cv	3.25	1.22	2.82	5.22	1.28	0.48	0.37	0.39	0.45	0.85	3.57	3.33	

Sd: Standard deviation, Cv: Coefficient of Variation, : lower monthly rainfall, : Higher monthly rainfall

3.2. Trend analysis: Result of the Mann-Kendall test

The MK test at monthly scale is used to evaluate the significance of trend in order to estimate whether or not the component can be neglected in the modeling process. The null hypothesis of no trend is tested for the p-value at a significance level of 5%. The null hypothesis has been accepted for all series. Therefore, trend in time series is not significant. Hence, trend components can be omitted from the mathematical model of the time series. Modeling is then focused on periodic and stochastic components. Results from the MK test are presented in the following table.

Table 4. - Results from the Mann-Kendall test

Station / *Variable*	*Saint-Louis*	*Bakel*	*Dakar*	*Kaolack*	*Ziguinchor*	*Tambacounda*
Recorded length (month)	492	492	492	492	492	492
Z statistic	0.301	2.530	1.519	0.161	1.855	-0.049
P Value	1.193	0.011	0.129	0.031	0.063	0.061
H_0	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
Conclusion	Trend free	Trend free	Trend free	Trend free	Trend free	Trend free

3.3. Evidencing seasonal behavior: analysis of autocorrelograms

The autocorrelogram analysis is done in the purpose of highlighting periodic behavior of the monthly rainfall series. Exploratory analysis of the autocorrelograms presented in Fig. 2, allows evidencing the periodic behavior of the observations. In addition, the whole autocorrelograms goes beyond the confidence band at significant level of 10%. Thus, the measured monthly rainfall are characterized by strong dependency between the within time series observations that leads to periodic behavior. Periodicity is characterized by succession of periodic peaks over the two periodic behavior (deterministic). Periodicity is characterized by succession of periodic peaks over the 12 months of the fundamental period as shown in Fig. 2.

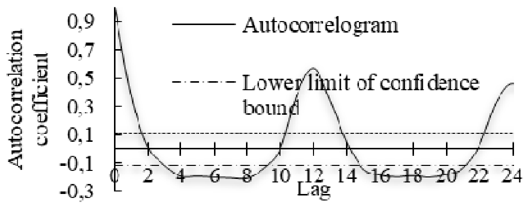


Fig. 2 (a) | Saint-Louis

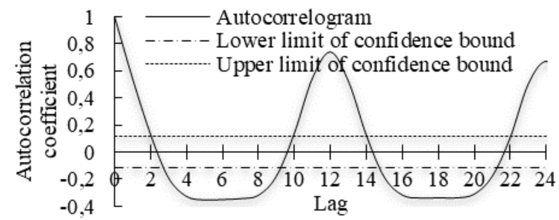


Fig. 2 (b) | Bakel

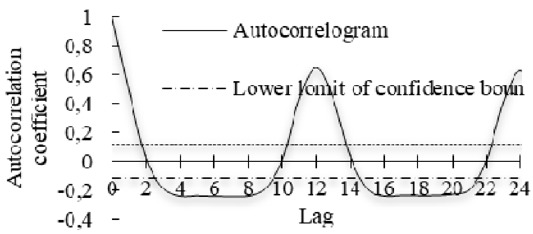


Fig. 2 (c) | Dakar

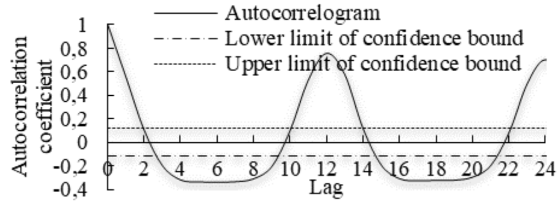


Fig. 2 (d) | Kaolack

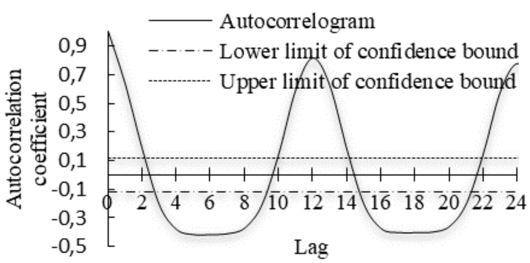


Fig. 2 (e) | Ziguinchor

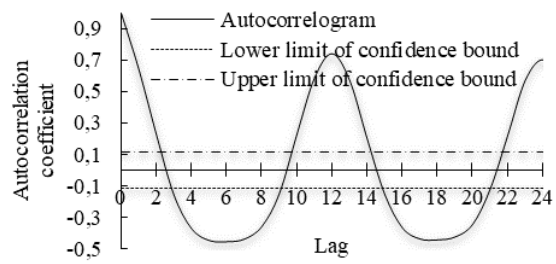


Fig. 2 (f) | Tambacounda

Figure2 : **Autocorrelogram of monthly rainfall in six selected raingauges**

3.4. Simulation of the seasonal component in the time series

In this section, the number of significant harmonics is estimated by the analysis of the cumulated periodograms shown in Fig.3. Significant harmonics are considered as the ones showing considerable gap in the periodogram (Kottegoda 1980; Bhakar 2006). The cumulative periodograms are obtained by plotting cumulative variances (Eq.06) in the harmonic analysis.

The exploratory analysis of the periodograms, shows that the significant harmonics are of four at Saint-Louis, Dakar and Kaolack and Bakel. At Tambacounda and Ziguinchor, tree significant harmonics have been retained. It is very important to notice that the steadier is the rainfall measurement, the lower the periodogram harmonics influencing

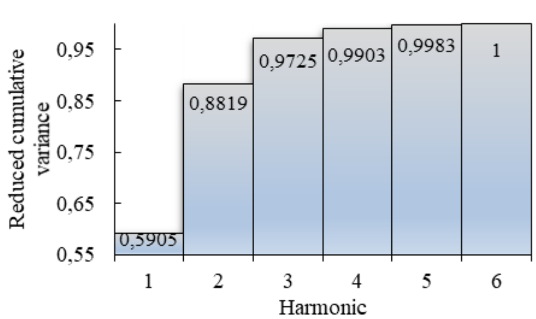


Fig. 3 (a) | Saint-Louis

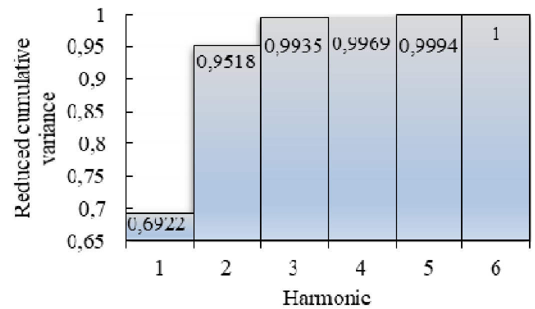


Fig. 3 (b) | Bakel

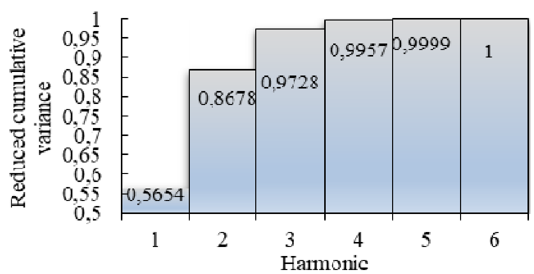


Fig. 3 (c) | Dakar

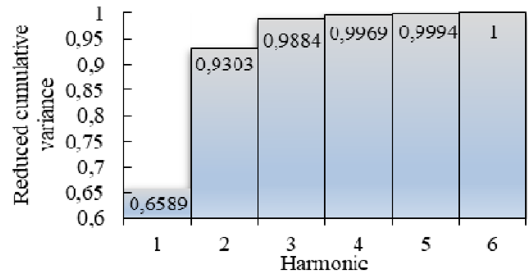


Fig. 3 (d) | Kaolack

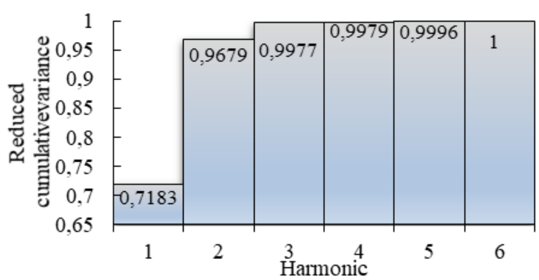


Fig. 3 (e) | Ziguinchor

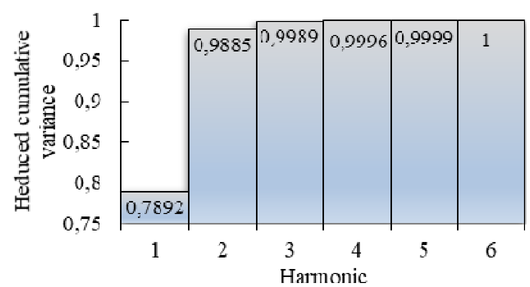


Fig. 3 (f) | Tambacounda

Figure3 : **Cumulative periodogram of monthly rainfall**

3.5. Modeling process

The retained models for the whole stations after estimating parameters of the basic equations are show by the following mathematical formula (Eq.18 to 23) including the harmonic coefficients, the estimated means and the autoregressive parameters. Their performing lead to results shown in Fig. 5 and 6. The statistical characteristic of the residual are shown in table 5 and associated autocorrelation in represented in Figure 4. Their variation coefficients are important in magnitude showing a random feature of this component (noise).

Saint-Louis

$$\hat{X}_t = 20.73 - 32.91\sin\frac{2\pi t}{12} - 11.05\cos\frac{2\pi t}{12} + 15.58\sin\frac{4\pi t}{12} - 18.76\cos\frac{4\pi t}{12} + 9.14\sin\frac{6\pi t}{12} + 10.06\cos\frac{6\pi t}{12} - 3.49\sin\frac{8\pi t}{12} + 4.92\cos\frac{8\pi t}{12} + 0.192\delta_{t-1} + \eta_t \quad \text{[Eq.18]}$$

Bakel

$$\hat{X}_t = 43.72 - 61.77\sin\frac{2\pi t}{12} - 40.25\cos\frac{2\pi t}{12} + 40.47\sin\frac{4\pi t}{12} - 20.00\cos\frac{4\pi t}{12} + 1.51\sin\frac{6\pi t}{12} + 18.03\cos\frac{6\pi t}{12} - 4.67\sin\frac{8\pi t}{12} + 2.2\cos\frac{8\pi t}{12} + 0.312\delta_{t-1} + \eta_t \quad \text{[Eq.19]}$$

Dakar

$$\hat{X}_t = 28.93 - 47.4\sin\frac{2\pi t}{12} - 18.90\cos\frac{2\pi t}{12} + 25.76\sin\frac{4\pi t}{12} - 27.00\cos\frac{4\pi t}{12} + 11.11\sin\frac{6\pi t}{12} - 18.97\cos\frac{6\pi t}{12} - 9.82\sin\frac{8\pi t}{12} + 3.02\cos\frac{8\pi t}{12} + 0.285\delta_{t-1} + \eta_t \quad \text{[Eq.20]}$$

Kaolack

$$\hat{X}_t = 48.81 - 73.97\sin\frac{2\pi t}{12} - 38.46\cos\frac{2\pi t}{12} + 43.44\sin\frac{4\pi t}{12} - 31.24\cos\frac{4\pi t}{12} + 5.79\sin\frac{6\pi t}{12} + 24.07\cos\frac{6\pi t}{12} - 9.46\sin\frac{8\pi t}{12} - 0.97\cos\frac{8\pi t}{12} + 0.318\delta_{t-1} + \eta_t \quad \text{[Eq.21]}$$

Ziguinchor

$$\hat{X}_t = 100.74 - 147.80\sin\frac{2\pi t}{12} - 35.10\cos\frac{2\pi t}{12} + 86.70\sin\frac{4\pi t}{12} - 50.91\cos\frac{4\pi t}{12} + 2.03\sin\frac{6\pi t}{12} + 34.65\cos\frac{6\pi t}{12} \; 0.382\delta_{t-1} + \eta_t \quad \text{[Eq.22]}$$

Tambacounda

$$\hat{X}_t = 59.73 - 78.75\sin\frac{2\pi t}{12} - 55.52\cos\frac{2\pi t}{12} + 45.09\sin\frac{4\pi t}{12} - 17.67\cos\frac{4\pi t}{12} - 0.61\sin\frac{6\pi t}{12} + 11.05\cos\frac{6\pi t}{12} + 0.403\delta_{t-1} + \eta_t \quad \text{[Eq.23]}$$

Table 5: Statistical characteristics of the residual components

Variables \ Stations	St. Louis	Bakel	Dakar	Kaolack	Ziguinchor	Tambacounda
Mean	-0.029	-0.014	-0.011	-0.017	-0.024	-0.017
Sd	28.51	36.67	32.06	38.13	61.32	38.10
Cv	983.10	-2913.63	-2914.54	-2242.94	-2555	-2241.17

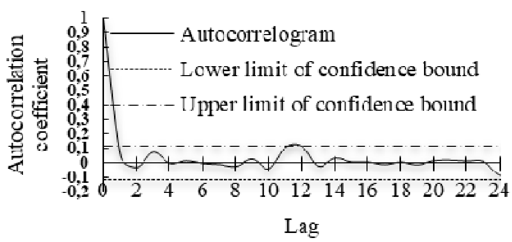


Fig. 4 (a) | Saint-Louis

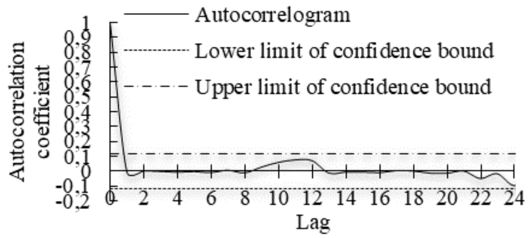


Fig. 4 (b) | Bakel

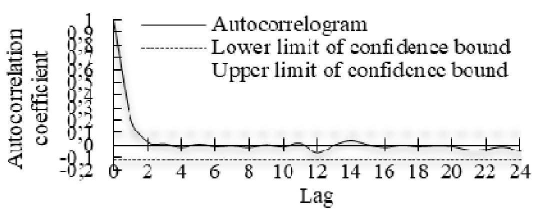


Fig. 4 (c) | Dakar

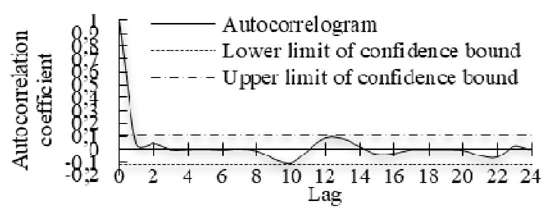


Fig. 4 (d) | Kaolack

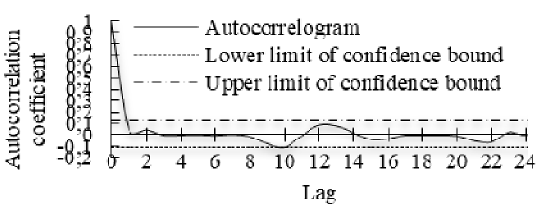


Fig. 4 (e) | Ziguinchor

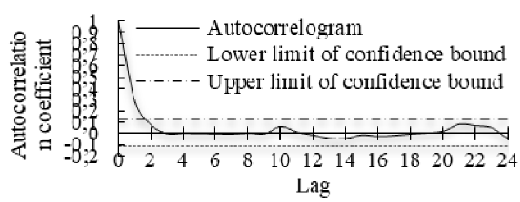


Fig. 4 (f) | Tambacounda

Figure4 : **Autocorrelogram of residual series of monthly rainfall in the six raingauges**

3.6 . Evolution of observed and simulated mean monthly rainfall

The analysis of observed mean monthly rains against simulated ones allows the evaluating of the retained models. In addition, comparison of the statistic characteristics of the observed and the simulated monthly series is also used (Table 6). Moreover, the analysis of the linear correlation between calculated mean monthly rainfall and observed ones with the coefficient of variation has been involved. Difference between the observed and the simulated mean monthly rainfall (absolute error) are of 0.01 mm for Saint-Louis and Ziguinchor, of 0.02 mm for Bakel and Dakar, and of 0.03 mm for Kaolack and Tambacounda. The coefficients of determination for linear correlation are of 0.988 for Saint-Louis, 0.9927 for Bakel, 0.995 for Dakar, 0.9975 for Kaolack, 0.9964 for Ziguinchor and 0.9996 for Tambacounda. Then, models for the mean monthly rainfall modeling are reliable. Simulated mean monthly time series fit observations of the whole studied sites. According to correlation coefficients, rainfall in the warm desert(Saint-Louis) and warm semi-arid zone (Dakar, Kaolack, Bakel and Tambacounda) are more unsteady (Fig.5.c,5.d,5.e,5.d) due to the high variability of local rainfall. Thus, model for simulating rainfall at the tropical savanna is naturally more skillful (Ziguinchor). Generally, results show that models represent acceptably the monthly rainfall in Senegal. Evolution in time of observed and simulated mean monthly rainfall time series are resented in Fig. 5. It has been noticed that the rainfall variability, the easier its modeling (Fig.5 e and 5 f)

Table 6: Characteristics (Mean, Standard deviation and coefficient of variation) of observed and simulated monthly rainfall

Station / Variable	Saint-Louis	Bakel	Dakar	Kaolack	Ziguinchor	Tambacounda
Measured mean monthly rainfall	20.73	43.72	28.93	48.81	100.7	59.73
Calculated mean monthly rainfall	20.74	43.74	28.95	48.84	100.8	59.76
Standard deviation of measured monthly rainfall	33.38	65.24	50.11	75.85	146.6	80.07
Standard deviation of calculated monthly rainfall	33.21	65.04	50.02	75.75	148.5	80.1
Coefficient of variation for measured monthly rainfall	1.61	1.49	1.73	1.55	1.45	1.34
Coefficient of variation for calculated monthly rainfall	1.60	1.48	1.72	1.55	1.47	1.34

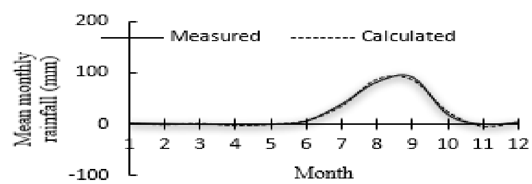


Fig. 5 (a) | Saint-Louis

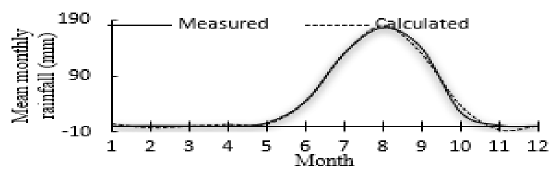


Fig. 5 (b) | Bakel

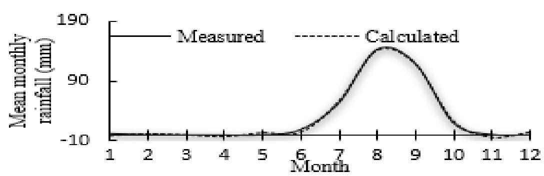


Fig. 5 (c) | Dakar

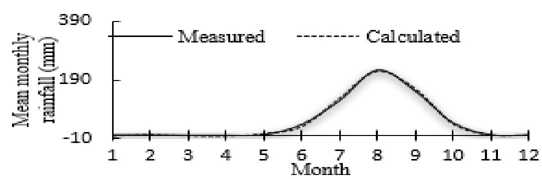


Fig. 5 (d) | Kaolack

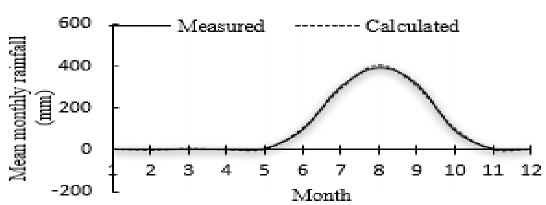


Fig. 5 (e) | Ziguinchor

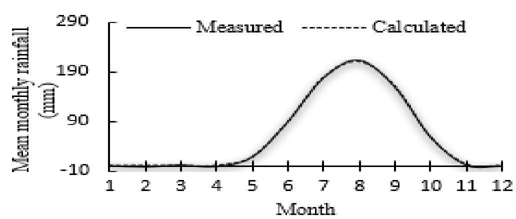


Fig. 5 (f) | Tambacounda

Figure5 : Variation of simulated and measured mean monthly rainfall

3.7. Evolution of observed and simulated monthly rainfall

The exploratory analysis of simulated against observed monthly rainfall and their statistical characteristics are used to evaluate the reliability of the models. The concerning characteristics are presented in Table 7. Deviation of the simulated monthly rainfall to observations over the time span (1970 to 2010) are of 2.75 mm for St-Louis, 2.66 mm for Bakel, 2.71 mm for Dakar, 1.55 mm for Kaolack, 1.4 mm for Ziguinchor and of 1.12 mm for Tambacounda. To improve visibility of the evolution in time of both simulated and observed monthly rainfall, two years of the entire period are represented according to locations: 1970 and 1980 for St-Louis, 1970 and 2010 for Bakel, 1980 and 1990 for Dakar, 1990 and 2000 for Kaolack, 1970 and 2000 for Ziguinchor and 1970 and 1990 for Tambacounda (Fig. 6). Results show that the model efficiency increases from the warm-desert zone to the tropical savanna zone (North to South). Linear correlations and determination coefficients confirm that models are reliable. Coefficients of variation are of about 0.9 (0.915 for Saint-Louis, 0.912 for Bakel and Dakar, 0.904 for Kaolack, 0.917 for Ziguinchor and 0.922 for Tambacounda. The evolution of simulated and observed monthly rainfall over the period of study combined to the linear fit curve and coefficient of determination are shown in Fig.7.

Table 7: Statistical characteristics of measured and calculated monthly rainfall

Station / Variable	St. Louis	Bakel	Dakar	Kaolack	Ziguinchor	Tambacounda
Measured monthly rainfall	20.73	43.72	28.93	48.81	100.7	59.73
Calculated monthly rainfall	23.48	46.38	31.64	50.36	102.1	60.85
Sd of measured monthly rainfall	42.78	72.32	58.33	82.7	154.9	87.43
Sd of calculated monthly rainfall	63.15	91.29	78.2	102.2	180.3	106.4

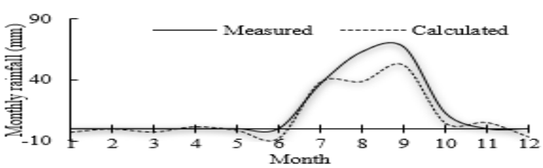


Figure 6 (a) | Saint-Louis 1970

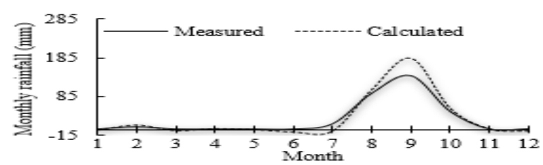


Figure 6 (b) | Saint-Louis 1980

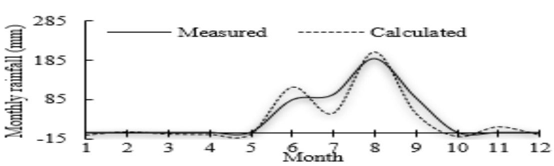


Figure 6 (c) | Bakel 1970

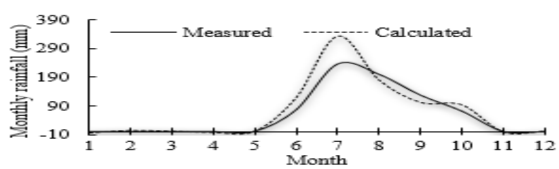


Figure 6 (d) | Bakel 2010

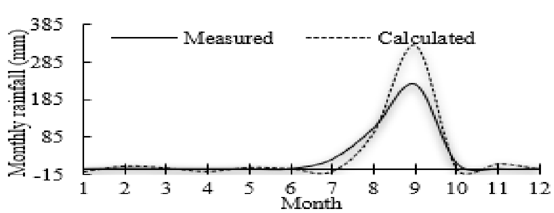


Figure 6 (e) | Dakar 1980

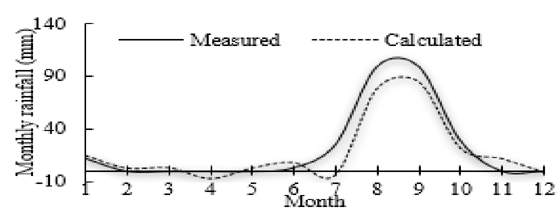


Figure 6 (f) | Dakar 1990

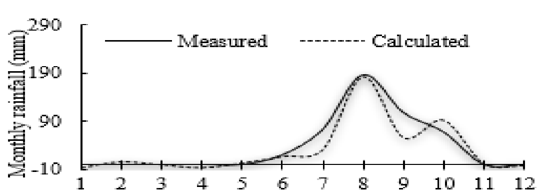


Figure 6 (g) | Kaolack 1990

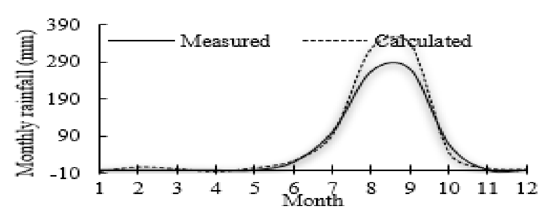


Figure 6 (h) | Kaolack 2000

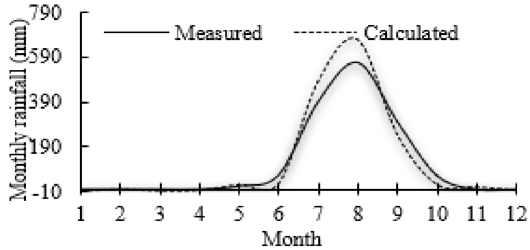


Figure 6 (i) | Ziguinchor 1970

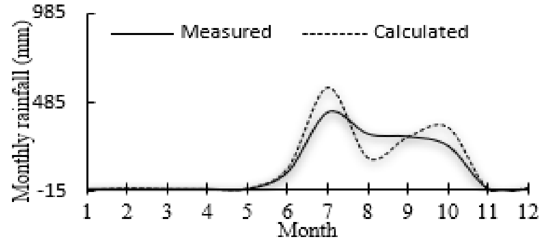


Figure 6 (j) | Ziguinchor 2000

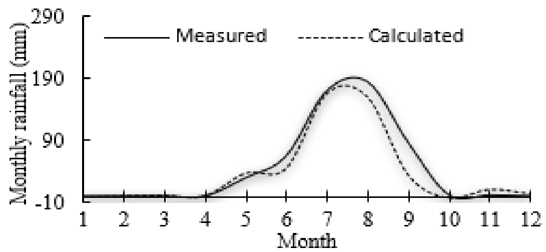


Figure 6 (k) | Tambacounda 1970

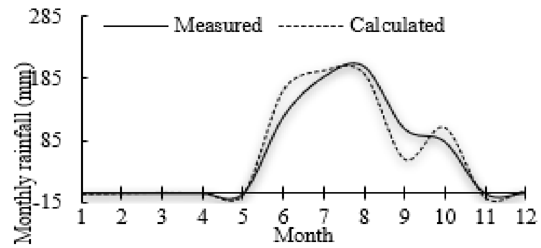


Figure 6 (l) | Tambacounda 1990

Figure6 : **Variation of generated and measured monthly rainfall**

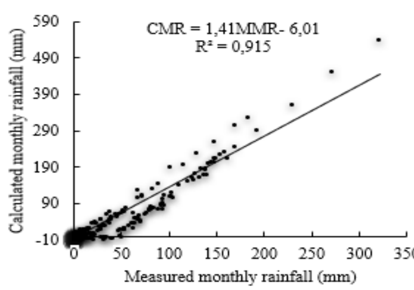


Figure 7 (a) | Saint-Louis

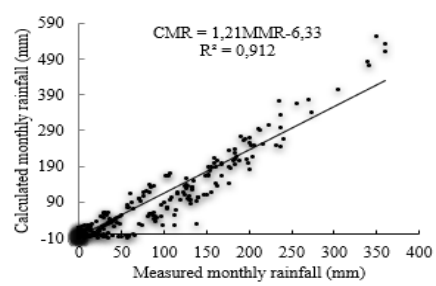


Figure 7 (b) | Bakel

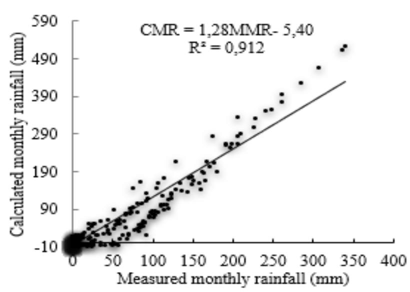


Figure 7 (c) | Dakar

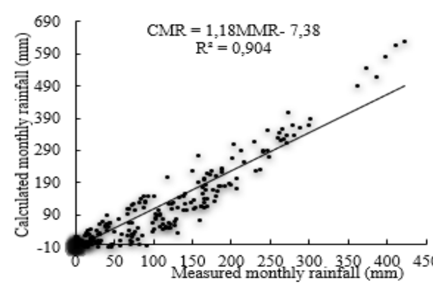


Figure 7 (d) | Kaolack

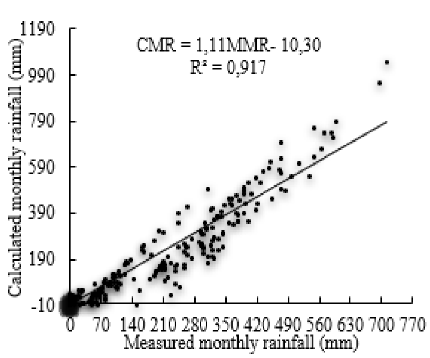


Figure 7 (e) | Ziguinchor

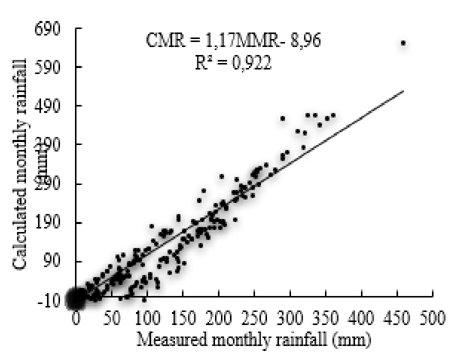


Figure 7 (f) | Tambacounda

Figure7 : **Relationship between generated and measured monthly rainfall**

4. Conclusion

Modeling of hydrocloclimatic variable is important for water resources management. It allows generating synthetic series usable in water resources planning and strategies designing towards adaptation to climate variability and change. In this study, reliable models capable of simulating monthly rainfall in the warm and desert (Saint-Louis), warm semi-arid (Dakar, Kaolack Bakel and Tambacounda), and tropical savanna (Ziguinchor) zones of Senegal are tested. The approach is based upon modeling separately components of the time series from a mathematical decomposition scheme (Trend, Periodic and Stochastic).

The MK test shows that the trend components can be neglected in the process, simplifying of the monthly rainfall modeling. The autocorrelogram analysis and associated test show a seasonal behavior of all the studied monthly rainfall. The seasonal component has been modeled using harmonic analysis in which significant harmonics are estimated through exploratory analysis of the corresponding periodograms. Significant harmonics vary from three at Ziguinchor and Tambacounda to four at the others locations. A first order of autoregressive process has been retained to fit stochastic components of the series. A comparison of the evolution in time between simulated and observed monthly rainfall is made to assess the skill of the models. Through this verification method, results are satisfactory. Further, for mean monthly rainfall simulation, coefficients of determination between simulated and observed mean monthly rains have reached 0.99. At monthly scale, the coefficients of variation range from 0.904 for the model at Kaolack to 0.922 for the retained model at Tambacounda. Therefore, models can be used to generate monthly rainfall time series through the Senegal area for different purposes. The modeling approach is less accurate for rainfall in the warm desert (Saint-Louis) and warm semi-arid (Dakar, Kaolack, Bakel and Tambacounda) where climate change effects and climate variability are more perceptible. Proposed modeling approach in this study is simple and is used at the HFML-UCAD to overcome shortcomings that are monthly rainfall related

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